

Number Sense Strand

Students' understanding of fractions, decimals, percents, and integers—and their relationship to each other and to the other disciplines of mathematics—is an essential component of their mathematics learning. CAHSEE test questions in the Number Sense strand require students to demonstrate a foundational understanding of numbers and ways they are represented.

Students will be asked to:

- solve problems with fractions, decimals, and percents
- compare and order numbers
- demonstrate an understanding of percents, including those less than 1 and greater than 100
- understand and meaningfully interpret large and small numbers in scientific notation
- use specific characteristics of numbers, such as multiples, factors, and primes
- use and represent integers as the basis for the comparison of quantities.

Essential to success in this CAHSEE strand is the student's understanding of the mathematical operations and the ways they are related to each other. This understanding includes:

- the meaning of arithmetic operations with fractions, decimals, and integers
- the associative and commutative properties of addition and multiplication
- the distributive property of multiplication over addition
- the understanding and use of inverse relationships of addition and subtraction, multiplication, and division
- finding square roots, squaring numbers, and using the inverse relationship between them.

Students also should possess computational fluency. They should be able to select appropriate methods and tools for computing with fractions and decimals; perform mental arithmetic; use algorithms for computing with fractions, decimals, and integers; use strategies for estimation and for judging the reasonableness of results; and be able to analyze and explain methods for solving problems with proportions.

The ten California academic content standards covered by the CAHSEE Number Sense strand are discussed in the following pages.

Strand	Number Sense (NS)	<p>The radius of the earth's orbit is 150,000,000,000 meters. What is this number in scientific notation?</p> <p>A 1.5×10^{-11}</p> <p>B 1.5×10^{11}</p> <p>C 15×10^{10}</p> <p>D 150×10^9</p>
Standard	7NS1.1	
	Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.	
Construct	Conceptual Understanding	M00213

Scientific notation is required knowledge in science and engineering because many numbers are either so large or so close to zero that there is no other convenient way to write them. CAHSEE test questions in this standard require students to demonstrate understanding of the basic concepts of scientific notation using approximations of very large and very small numbers. Test questions may also involve the translation of approximate numbers into scientific notation, the comparison of numbers in scientific notation with either positive or negative exponents, and the understanding of the relative size of two numbers in scientific notation.

Sample Test Question

The sample question gives the radius of the earth's orbit as 150,000,000,000 meters and asks students to translate that number into scientific notation. The correct answer is choice B. Students should recognize that the place-value distance from the 1 (highest place value, 100 billion) to the decimal is 11 digits and that this value provides an appropriate representation of the equivalence as 10^{11} , also equivalent to $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$. Students also should know that expressions in scientific notation may include a multiplier, between 1 and 10, along with an exponential value of 10. While exact powers of 10 are expressed without a multiplier (e.g., 10^{11}), numbers such as 150,000,000,000 require a multiplier along with the equivalent power of 10. A typical method of finding the multiplier and the exponent is to count the number of decimal places the decimal must move to create a number between 1 and 10. In the example, the decimal point is moved 11 places to the left to get 1.5 for the multiplier and +11 for the exponent.

Analysis of Distractors

Two of the distractors in the sample test question, C and D, represent common errors students may make in finding equivalence between standard and scientific notation: Both distractors are equivalent in number to 150,000,000,000 (and 1.5×10^{11}), but neither has a multiplier within the appropriate range (from 1 to 10). Distractor A expresses the power of 10 as $\frac{1}{10^{11}}$ (until students have a clear understanding of the magnitude of very large and very small numbers, they are often unsure which direction the decimal point should move.)

Strand	Number Sense (NS)	<p>Which of the following numerical expressions results in a negative number?</p> <p>A $(-7) + (-3)$</p> <p>B $(-3) + (7)$</p> <p>C $(3) + (7)$</p> <p>D $(3) + (-7) + (11)$</p> <p>M00116</p>
Standard	7NS1.2	
	Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.	
Constructs	Procedural Skills, Conceptual Understanding	

All students should understand the basic arithmetic functions involving rational numbers in all forms and be comfortable performing calculations with positive and negative numbers. CAHSEE test questions in this standard require students to demonstrate computational fluency with rational numbers and an understanding of the relationships between these types of numbers. These skills are fundamental to achievement of the California academic content standards in mathematics.

Sample Test Question

The sample question asks students to compute four combinations of values with different signs and then identify which of the sums is negative. The correct answer is choice A. Students should understand that adding rational numbers with unlike signs requires finding the difference of their absolute values and then choosing the sign of the addend with the greater absolute value. Students should also recognize that adding rational numbers with like signs requires adding the absolute values and keeping the sign, as in choice A: $(-7) + (-3) = -10$.

Analysis of Distractors

The distractors present students with errors in computing with integers. The addends in two of the distractors, B and C, have the same absolute values as the addends in the correct answer, A, with sums of 4 and 10 respectively, but they are both positive integers. Distractor D introduces a third value and has a sum of 7, but it, too, is a positive integer.

Strand	Number Sense (NS)	<p>Some students attend school 180 of the 365 days in a year. About what part of the year do they attend school?</p> <p>A 18%</p> <p>B 50%</p> <p>C 75%</p> <p>D 180%</p>
Standard	7NS1.3	
Constructs	Convert fractions to decimals and percents and use these representations in estimations, computations and applications. Procedural Skills, Conceptual Understanding	

M00047

Students using mathematics in their daily lives will need to know how to convert decimals to fractions to percents with ease. The recognition of equivalent forms is essential for student fluency with numbers. CAHSEE test questions in this standard require students to demonstrate facility in finding equivalent values and representations for numbers as well as to find and compute values using fractions, decimals, and percents. Students must also be able to use these various representations for estimating and performing computations in mathematical applications.

Sample Test Question

The sample question asks students to determine what percent 180 is of 365. The correct answer is choice B. Students should recognize that the correct value is found by dividing the part (180) by the whole (365), giving $180 \div 365 \approx 0.493$. The question also requires conversion of 0.493 to a percent by multiplying by 100 ($0.493 = 49.3\%$). Finally, students should recognize that the phrase “about what part” in the stem calls for a rounded percent rather than an exact value, and that $49.3\% \approx 50\%$. It is to be expected that many students will use mental estimation skills, rather than computation, to determine that 180 is about half of 365.

Analysis of Distractors

Two of the distractors, A and D, represent a misunderstanding of 180 as 18% and 180%.

Distractor C indicates incorrect computation of the decimal equivalent of $\frac{180}{365}$ or a possible guess with apparent plausibility.

Strand	Number Sense (NS)	<p>The cost of an afternoon movie ticket last year was \$4.00. This year an afternoon movie ticket costs \$5.00. What is the percent increase of the ticket from last year to this year?</p> <p>A 10%</p> <p>B 20%</p> <p>C 25%</p> <p>D 40%</p>
Standard	7NS1.6	
Constructs	Procedural Skills, Conceptual Understanding	

M02158

CAHSEE test questions in this standard require students to demonstrate understanding of percent increase and decrease, which is a fundamental tool in analyzing numerical information. For example, a price change of one dollar can be very meaningful in terms of buying a loaf of bread and inconsequential in terms of buying a car. Students should understand that percent change clarifies the impact of this kind of change, and they should be able to calculate the change with facility. Standard 7NS1.6 will also be assessed with test questions that require students to find the percent decrease. Finding the quantity that results from a given percentage increase or decrease is assessed in standard 7NS1.7.

Sample Test Question

The sample question requires students to calculate the percent of increase for a movie ticket whose price increased from \$4.00 to \$5.00. The correct answer is choice C. Students should understand that finding the percent increase or decrease of a quantity requires first finding the difference between the initial value and the final value. In the sample question, the difference is \$1.00. Then to find the percent increase, students must know to compare the difference to the initial cost by using division: $\frac{\$1.00}{\$4.00}$. Finally, the resulting decimal must be converted to its equivalent percent: $1 \div 4 = 0.25 = 25\%$.

Analysis of Distractors

The distractors represent errors resulting from a misunderstanding of the concept being tested. Distractor A results from taking the difference between the two ticket prices and incorrectly converting it to a percent. Distractor B results from multiplication of the two values in the problem, \$4.00 and \$5.00. Distractor D results from converting the original ticket price to a percent.

Strand	Number Sense (NS)	Sally puts \$200.00 in a bank account. Each year the account earns 8% simple interest. How much interest will be earned in three years? A \$16.00 B \$24.00 C \$48.00 D \$160.00	M02119
Standard	7NS1.7		
	Solve problems that involve discounts, markups, commissions, and profit, and compute simple and compound interest.		
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving		

CAHSEE test questions in this standard require students to solve a variety of problems involving percents. Both consumers and people working in business need to understand the mathematical meaning of common business terms such as commission, profit, and loss, as well as how to make interest computations. Solving problems of these types is one of the most important skills students need as they become adults. Understanding these concepts and their applications can mean the difference between students managing their money and other resources well, or not at all. This standard will also be assessed with test questions that require students to find simple and compound interest, as well as discounts, markups, and commissions. A maximum of three iterations is used for questions that involve calculating compound interest. The iterations include the initial multiplication of principal by interest rate.

Sample Test Question

The sample question asks students to determine the amount of simple interest \$200 will earn in three years at the given rate. The correct answer is choice C. Students should recognize that simple interest is calculated by multiplying the principal by the annual rate and then multiplying by the time. In the sample question, the principal is \$200, the rate is 8%, and the time is 3 years. To calculate correctly, students are also required to convert 8% to its decimal equivalent ($\$200 \times 0.08 \times 3 = \48).

Analysis of Distractors

The distractors represent errors resulting from failure to perform one of the required steps and/or from a computation error. Distractor A represents one year's interest and results from the multiplication of the principal and the interest rate only. Distractor B represents multiplication of the rate times the number of years only. Distractor D may be attractive to students who converted the interest rate to a decimal incorrectly, multiplying \$200 by 0.8, and also failed to multiply by the number of years.

Strand	Number Sense (NS)	$\frac{10^{-2}}{10^{-4}} =$
Standard	7NS2.1	A 10^{-6}
	Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.	B 10^{-2}
		C 10^2
Constructs	Procedural Skills, Conceptual Understanding	D 10^8
		M02832

CAHSEE test questions in this standard require students to understand the concept of negative exponents. One of the most powerful concepts in mathematics is that exponential notation can be extended to include new concepts. Negative exponents are an example of this kind of extension.

Sample Test Question

The sample question presents a numerical calculation requiring students to demonstrate their understanding of the rule for dividing expressions involving exponents with a common base. The correct answer is choice C. Students should understand that 10^{-2} is equivalent

to $\frac{1}{10^2}$ and $\frac{1}{10^{-4}}$ is equivalent to 10^4 . Thus, it is possible to represent the problem

as $\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10}$, making the underlying concept more apparent. $\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10}$ can be reduced

by dividing it by $\frac{10 \cdot 10}{10 \cdot 10}$ to $10 \cdot 10$, which is equivalent to 10^2 . Once the students master the

concept behind negative exponents, they understand that when dividing exponential expressions with the same base, the exponents must be subtracted, so that $\frac{10^{-2}}{10^{-4}}$ is equivalent

to $10^{(-2)-(-4)}$, which is equivalent to 10^2 .

Analysis of Distractors

The distractors represent misunderstandings of the concepts involved in the calculation.

Distractor A represents the addition of the exponents or an error in the subtraction of $(-4) - (-2)$. Distractor B shows a failure to apply the negative signs correctly in the same subtraction. Distractor D represents a failure to understand the fundamental concept, as it results from multiplying -2 by -4 .

Strand	Number Sense (NS)	Which fraction is equivalent to $\frac{5}{6} + \frac{7}{8}$?
Standard	7NS2.2 Add and subtract fractions by using factoring to find common denominators.	A $\frac{35}{48}$
Constructs	Procedural Skills, Conceptual Understanding	B $\frac{6}{7}$
		C $\frac{20}{21}$
		D $\frac{41}{24}$

M12713

The focus of this content standard is on the students' ability to add and subtract fractions with unlike denominators that share one or more factors. Students should be able to find the prime factorization of each denominator, then combine factors to determine the least common denominator. CAHSEE test questions in this standard require students to perform addition and subtraction arithmetic using equivalent fractions with common denominators. The algorithmic approach of this standard is associated with the requirement that common denominators be determined by factoring.

Sample Test Question

The sample requires students to find the needed common denominator for 6 and 8 using prime factors. The correct answer choice is D. Students should recognize that the prime factors for 6 are 2 and 3 and that the prime factors for 8 are 2 and 4. Since the common prime factor is 2, including the additional factors of 3 and 4 gives $2 \times 3 \times 4$ as the prime factors of the least common denominator. Students should then find equivalent fractions using the least common denominator and add the fractions.

Analysis of Distractors

The distractors represent misunderstandings of the concept being tested. Distractor A shows the numerators and denominators being multiplied together, respectively. Distractor B shows the numerators and denominators being added together, respectively. Distractor C represents the numerators of the equivalent fractions being expressed as a numerator and denominator.

Strand	Number Sense (NS)	$(3^8)^2 =$
Standard	7NS2.3	A 3^4
Multiply, divide, and simplify rational numbers by using exponent rules.		B 3^6
		C 3^{10}
Constructs	Procedural Skills, Conceptual Understanding	D 3^{16}

M02406

CAHSEE test questions in this standard require students to select the appropriate rules for operations with exponents with common bases and perform accurate computations in simplifying rational numbers. Students should understand the following rules:

- adding exponents when multiplying numbers with common bases
- subtracting exponents when dividing numbers with common bases
- multiplying exponents when raising a number to a particular power.

Test questions may include those requiring multi-step operations, such as the simplification of numerators and denominators with common factors.

Sample Test Question

The sample question requires students to expand $(3^8)^2$ using the rule for multiplying exponents in parentheses $\left[(a^b)^c = a^{bc}\right]$. The correct answer is choice D. In this instance, students should use the rule to determine that $(3^8)^2 = 3^{16}$.

Analysis of Distractors

The distractors present the other three operations that could be performed. Distractor A represents the inappropriate operation of division, rather than multiplication. Distractor B represents the inappropriate operation of subtraction. Distractor C represents the addition of the exponents, which students might choose if they confused this calculation with one requiring multiplication of exponential expressions with the same base.

Strand	Number Sense (NS)	The square root of 150 is between
Standard	7NS2.4	A 10 and 11.
Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.		B 11 and 12.
		C 12 and 13.
		D 13 and 14.
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving	M02666

CAHSEE test questions in this standard require students to demonstrate a conceptual understanding of powers and roots and their inverse relationship. The idea of mathematical inverse is a key precursor for algebraic reasoning, and students should understand that taking a root is the inverse operation of raising a number or expression to a power. For example, students should know that $\sqrt{9^2} = 9$. Students will not be required to calculate the square root for a number that is not a perfect square, but students should be able to approximate the value of the square root of an integer that is not a perfect square.

Sample Test Question

The sample question asks students to find an approximation of the square root of 150, an integer that is not a perfect square. The correct answer is choice C. Students should recognize that they must first determine both the closest perfect square greater than the given integer and the closest perfect square less than the integer. This task may involve some trial and error multiplication along with the application of knowledge of squares and square roots. Since the number 150 is not a perfect square, students may recall or calculate that 144 is a perfect square and that 169 is the next perfect square. Since 144 is close to but less than 150 and 169 is close to but greater than 150, the square root of 150 must lie between those two perfect squares.

Analysis of Distractors

The distractors misplace 150 between other numbers and may be selected by students who do not understand the concept of square root or who may incorrectly calculate the square of one of the numbers.

Strand	Number Sense (NS)	<div>If $x = 3$, what is the value of x?</div> <div>A -3 or 0</div> <div>B -3 or 3</div> <div>C 0 or 3</div> <div>D -9 or 9</div> <div>M02122</div>
Standard	7NS2.5	
<div>Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.</div>		
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving	

CAHSEE test questions in this standard require students to demonstrate a conceptual understanding of absolute value and its meaning as represented on a number line. Relating absolute value to distance on the number line may help students understand the concept: Distance cannot be negative, but there will always be two numbers on the number line that are the same distance from zero. Questions may require students to find the absolute value after performing a basic computation.

Sample Test Question

The test question asks students to determine the possible values for x in a simple absolute value equation. The correct answer is choice B. Students should recognize that since the absolute value of a number is the distance on a number line from that number to zero in either direction, all absolute values are positive numbers: $|x| = x$ and $|-x| = x$. In the test question, if $|x| = 3$, then $x = -3$ or $x = 3$, since $|3| = 3$ and $|-3| = 3$.

Analysis of Distractors

The distractors represent misunderstandings of the concept and notation for absolute value. Distractors A and C incorrectly equate the absolute value of 3 with 0 and offer either a negative or a positive value of 3. Distractor D correctly provides both a negative and a positive value but inappropriately associates the absolute value of 3 with the square of 3.